European Network of
Transmission System Operators
for Electricity

Phase Shift Transformers Modelling

Version: 1.0.0

CGMES v2.4.14

28 May 2014

Contents

Contents....2
1. Introduction....4
2. Mapping to CIM classes and attributes....4
3. Reactance formulas summary table....5
4. Symmetrical Phase shifters....6
4.1. One phase diagram and equations....6
4.2. Expression of the angle and ratio per tap....6
4.3. Expression of the equivalent series reactance given the angle....6
4.4. Three-phase diagrams....7
5. Quadrature booster....8
5.1. One phase diagram and equations....8
5.1. Expression of the angle and ratio per tap....8
5.2. Expression of the equivalent series reactance given the angle....9
5.3. Three-phase diagrams....9
6. Asymmetrical Phase Shifter....10
6.1. One phase diagram and equations....10
6.2. Expression of the angle and ratio per tap....10
6.3. Expression of the equivalent series reactance given the angle....10
6.4. Three-phase diagram....11
7. In-phase transformer and symmetrical phase shifter....11
7.1. One phase diagram and equations....11
7.2. Expression of the angle and ratio per tap....12
7.3. Expression of the equivalent series reactance given the angle and the in-phase transformer ratio....12
7.4. Technology principles....12
8. In-phase transformer and asymmetrical phase shifter....13
8.1. One phase diagram and equations....13
8.2. Expression of the angle and ratio per tap....13
8.3. Expression of the equivalent series reactance given the angle and the in-phase transformer ratio....13
8.4. Technology principles....14
9. Appendix....15
9.1. Symmetrical Phase Shifters with two cores....15
9.2. Quadrature boosters....18
9.3. Quadrature booster with two cores....18
9.4. Quadrature booster with a single core....20

ENTSO-E AISBL • Avenue de Cortenbergh 100 • 1000 Brussels • Belgium • Tel + 32 2 741 09 50 • Fax + 32 2 741 09 51 • info@entsoe.eu • www. entsoe.eu

9.5. Asymmetrical phase shifter 22
9.6. Asymmetrical phase shifter with two cores 22
9.7. Asymmetrical phase shifter with a single core 24
9.8. In-phase transformer and asymmetrical phase shifter 26

ENTSO-E AISBL • Avenue de Cortenbergh 100 • 1000 Brussels • Belgium • Tel + 32 2 741 09 50 • Fax + 32 2 741 09 51 • info@entsoe.eu • www. entsoe.eu

1. Introduction

This document describes Phase-Shifting Transformer (PST) modelling according to the type and technology of the equipment; and more specifically, equivalent series reactance of phase shifters in the positive sequence as a function of the phase shift angle. In these specific models, the resistances and the magnetizing currents are always neglected.

This document also provides the mapping between the equations for each type and technology of PST to the CIM classes and attributes which are included in the ENTSO-E Common Grid Model Exchange Standard (CGMES) version 2.4.

2. Mapping to CIM classes and attributes

To illustrate how to use the PST modelling described in this document, the mapping to CIM 16 classes is presented in the following table:

PST type	CIM class
General Case	**cim:PhaseTapChangerTabular**
Symmetrical phase shifters	cim:PhaseTapChangerSymmetrical or cim:PhaseTapChangerLinear
Asymmetrical phase shifter	cim:PhaseTapChangerAsymmetrical
In-phase transformer and symmetrical phase shifter	cim:PhaseTapChangerSymmetrical and cim:RatioTapChanger
In-phase transformer and asymmetrical phase shifter	cim:PhaseTapChangerSymmetrical and cim:RatioTapChanger

Recommendation

- It is highly recommended to use **tabular data** to exchange PST parameters (cim:PhaseTapChangerTabular) instead of having to recalculate the parameters per tap according to each model type

PST attribute	CIM attribute
n	cim:PhaseTapChangerTablePoint.step
n_0	cim:TapChanger.neutralStep
δu	cim:PhaseTapChangerNonLinear.voltageStepIncrement
r	cim:TapChangerTablePoint.ratio
α	cim:PhaseTapChangerTablePoint.angle
$\delta\alpha$	cim:PhaseTapChangerLinear.stepPhaseShiftIncrement
$X(\alpha)$	cim:TapChangerTablePoint.x
$X(0)$	cim:PhaseTapChangerLinear.xMin or

	cim:PhaseTapChangerNonLinear.xMin
$X(\alpha_{\max})$	cim:PhaseTapChangerLinear.xMax or cim:PhaseTapChangerNonLinear.xMax
θ	cim:PhaseTapChangerAsymmetrical.windingConnectionAngle

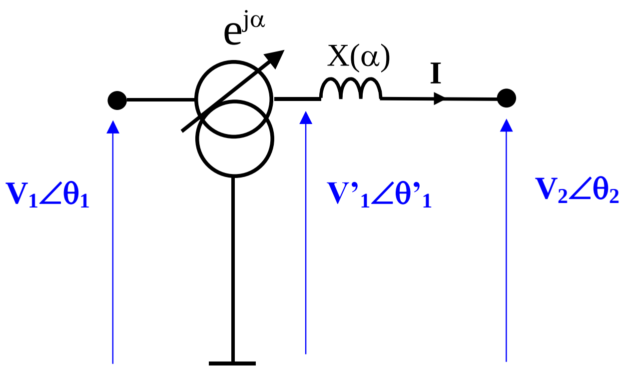
3. Reactance formulas summary table

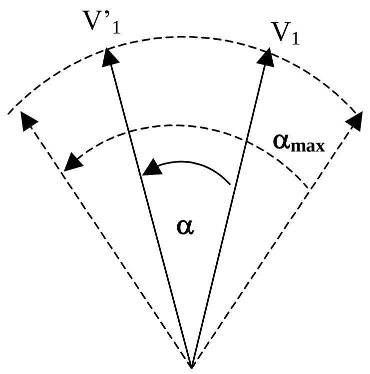
Equipment type	Equivalent series reactance as a function of the phase shift angle
Symmetrical Phase shifters	$X(\alpha) = X(0) + \left(X(\alpha_{\max}) - X(0)\right)\left(\frac{\sin(\alpha/2)}{\sin(\alpha_{\max}/2)}\right)^2$
Asymmetrical Phase Shifter	$X(\alpha) = X(0) + \left(X(\alpha_{\max}) - X(0)\right)\left(\frac{\tan\alpha}{\tan\alpha_{\max}} \frac{\sin\theta - \tan\alpha_{\max}\cos\theta}{\sin\theta - \tan\alpha\cos\theta}\right)^2$
In-phase transformer and symmetrical phase shifter	$X(r,\alpha) = X_r(r_{nom})\left(\frac{r}{r_{nom}}\right)^2 + X_\alpha(0) + \left(X_\alpha(\alpha_{\max}) - X_\alpha(0)\right)\left(\frac{\sin(\alpha/2)}{\sin(\alpha_{\max}/2)}\right)^2$
In-phase transformer and asymmetrical phase shifter	$X(r,\alpha) = X_r(r_{nom})\left(\frac{r}{r_{nom}}\right)^2 + X_\alpha(0)$ $+\left(X_\alpha(\alpha^0_{\max}) - X_\alpha(0)\right)\left(\frac{\tan\alpha}{\tan\alpha_{\max}(r)} \frac{\sin\theta - \tan\alpha_{\max}(r)\cos\theta}{\sin\theta - \tan\alpha\cos\theta}\right)^2$ with $\alpha_{\max}(r) = A\tan\left(\frac{\sin\theta}{\frac{r}{r_{nom}\tan\alpha^0_{\max}}\left(\sin\theta - \tan\alpha^0_{\max}\cos\theta\right) + \cos\theta}\right)$ and $\alpha^0_{\max} = \alpha_{\max}(r_{nom})$

Variable	Meaning
$\alpha_{\max}$	Maximal phase shift: the maximum angle value of the phase shifter angles table
r_{nom}	Nominal ratio of the in-phase transformer
$\alpha^0_{\max}$	Maximal phase shift when the in-phase transformer ratio is nominal

4. Symmetrical Phase shifters

4.1. One phase diagram and equations





$$V2 = V1e^{j\alpha} - jX(\alpha)I \qquad\qquad V'1 = V1\left(1+2j\sin\frac{\alpha}{2}e^{j\frac{\alpha}{2}}\right) = e^{j\alpha}V1$$

4.2. Expression of the angle and ratio per tap

Based on the figure above:

$$\alpha = (n-n_0)\cdot\delta\alpha \qquad \text{or} \qquad \alpha = 2A\tan\left(\frac{(n-n_0)\cdot\delta u}{2}\right)$$

$$r = 1$$

4.3. Expression of the equivalent series reactance given the angle

Assuming the reactance of the regulating winding varies as the square of the number of turns, the equivalent reactance can be written as follows for non delta-hexagonal technologies (see proof in appendix 9.1):

$$X(\alpha) = X(0) + \left(X(\alpha_{\max}) - X(0)\right)\left(\frac{\sin(\alpha/2)}{\sin(\alpha_{\max}/2)}\right)^2$$

3 parameters:

- α_{max}: maximal phase shift
- X(0): equivalent series reactance at zero phase shift $\alpha=0$
- $X(\alpha_{max})$: equivalent series reactance at maximal phase shift $\alpha=\alpha_{max}$

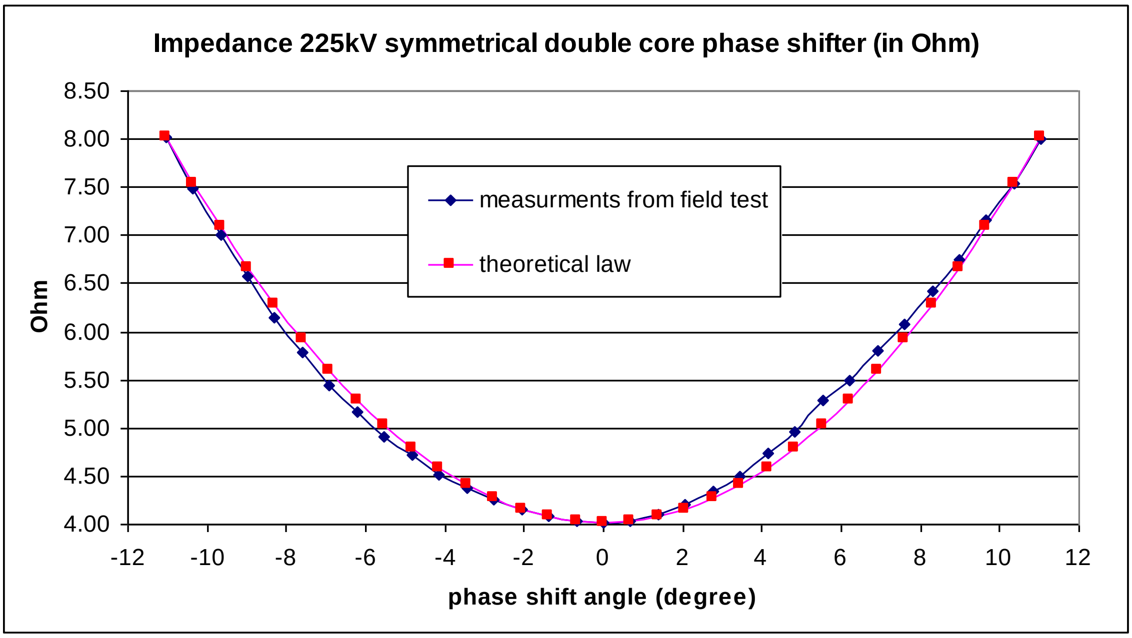
1 variable:

- α: current phase shift

The formula above is valid for single or double core symmetrical phase shifters except for the hexagonal technology.

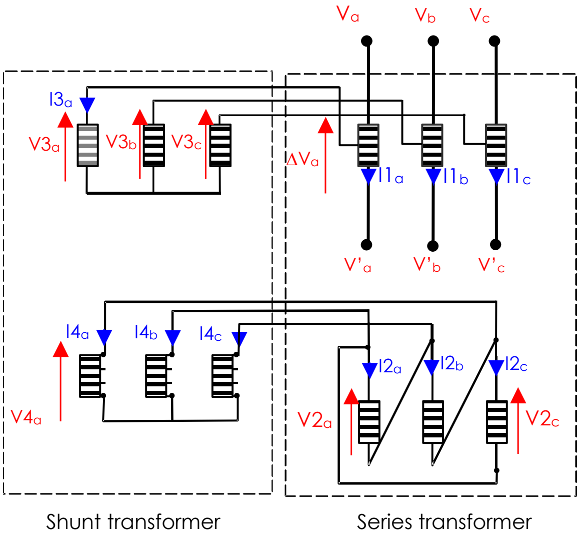
For single core symmetrical phase shifters: X(0)=0

ENTSO-E AISBL • Avenue de Cortenbergh 100 • 1000 Brussels • Belgium • Tel + 32 2 741 09 50 • Fax + 32 2 741 09 51 • info@entsoe.eu • www. entsoe.eu

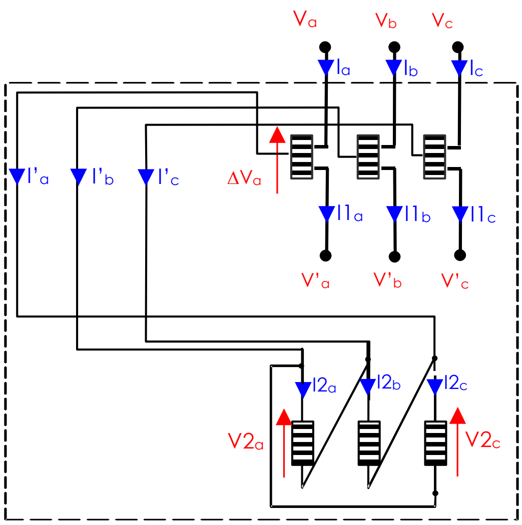


4.4. Three-phase diagrams

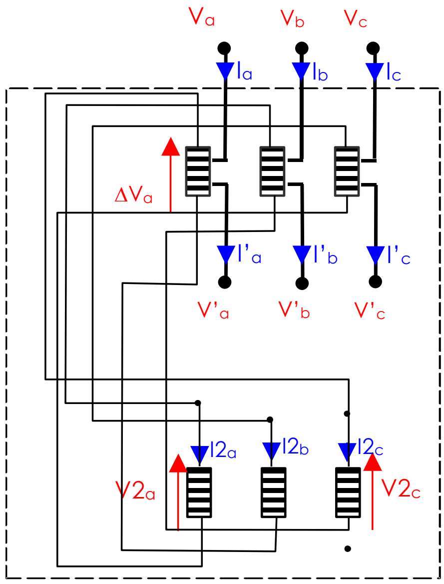
Dual core:



Single core:

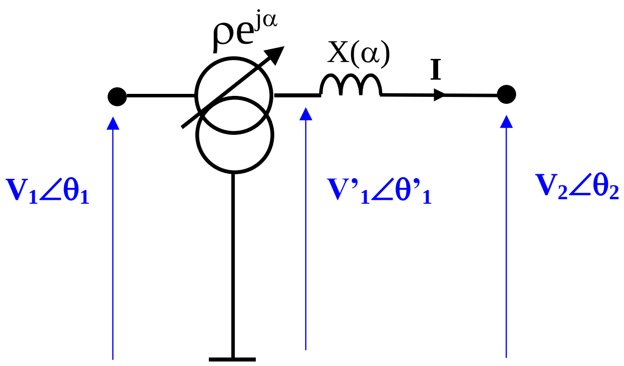


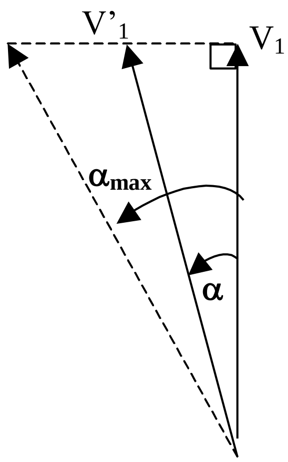
Single core, delta hexagonal:



5. Quadrature booster

5.1. One phase diagram and equations





$$V'1 = V1(1 + j\tan\alpha) = \rho e^{j\alpha} V1$$

$$V2 = V1\rho e^{j\alpha} - jX(\alpha)I$$

ρ and α vary with ρ.cos α = constant.

5.1. Expression of the angle and ratio per tap

Based on the figure above:

$$\alpha = A\tan((n - n_0)\cdot \delta u)$$

$$r = \frac{1}{\sqrt{((n - n_0)\cdot \delta u)^2 + 1}}$$

5.2. Expression of the equivalent series reactance given the angle

Assuming the reactance of the regulating winding varies as the square of the number of turns, the equivalent reactance of the quadrature booster can be written as follows (see proof in appendix 9.2):

$$X(\alpha) = X(0) + \left(X(\alpha_{max}) - X(0)\right)\left(\frac{\tan(\alpha)}{\tan(\alpha_{max})}\right)^2$$

3 parameters:

- α_{max}: maximal phase shift
- X(0): equivalent series reactance at zero phase shift
- $X(\alpha_{max})$: equivalent series reactance at maximal phase shift

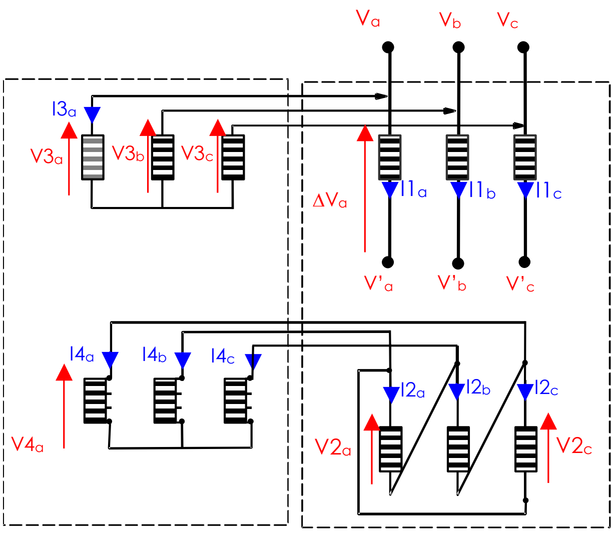
1 variable:

- α: current phase shift

For quadrature boosters with a single core: X(0)=0

5.3. Three-phase diagrams

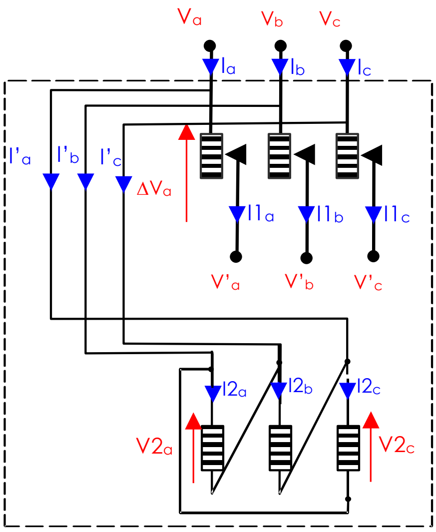
Dual core:



Shunt transformer

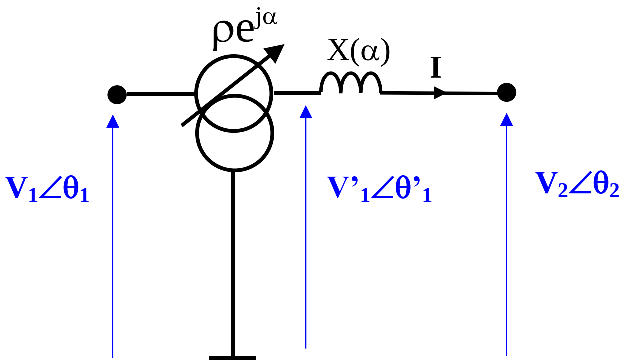
Series transformer

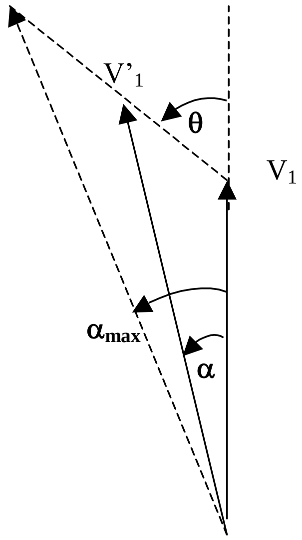
Single core:



6. Asymmetrical Phase Shifter

6.1. One phase diagram and equations





$$V'1 = V1\left(1 + e^{j\theta}.\frac{\tan\alpha}{\sin\theta - \tan\alpha\cos\theta}\right) = \rho e^{j\alpha}V1$$

$$V2 = V1.\rho e^{j\alpha} - jX(\alpha)I$$

θ is fixed, only α and ρ vary.

6.2. Expression of the angle and ratio per tap

Based on the figure above:

$$\alpha = A\tan\left(\frac{(n-n_0)\cdot\delta u\cdot\sin\theta}{1+(n-n_0)\cdot\delta u\cdot\cos\theta}\right)$$

$$r = \frac{1}{\sqrt{((n-n_0)\cdot\delta u\cdot\sin\theta)^2 + (1+(n-n_0)\cdot\delta u\cdot\sin\theta)^2}}$$

6.3. Expression of the equivalent series reactance given the angle

Assuming the reactance of the regulating winding varies as the square of the number of turns, the equivalent reactance can be written as follows (see proof in appendix 9.5):

$$X(\alpha) = X(0) + (X(\alpha_{max}) - X(0))\left(\frac{\tan\alpha}{\tan\alpha_{max}}\frac{\sin\theta - \tan\alpha_{max}\cos\theta}{\sin\theta - \tan\alpha\cos\theta}\right)^2$$

4 parameters:

- α_{max}: maximal phase shift

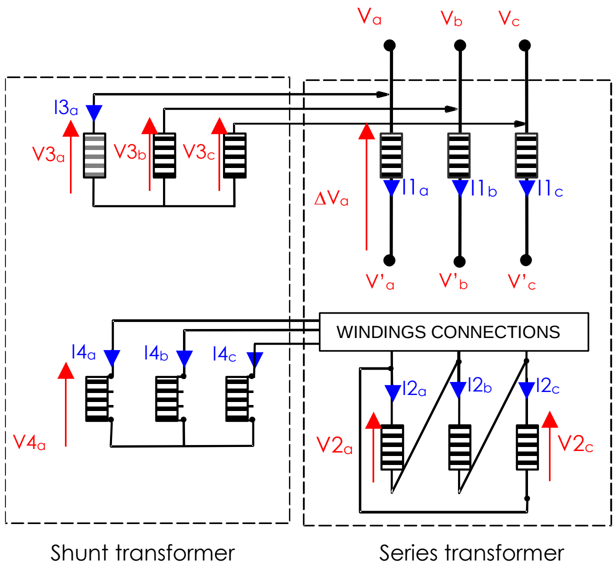
- X(0): equivalent series reactance at zero phase shift
- $X(\alpha_{max})$: equivalent series reactance at maximal phase shift
- θ: boost voltage angle

1 variable:

- α: current phase shift

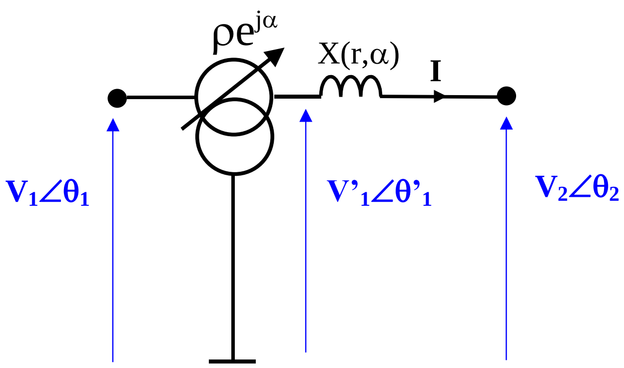
6.4. Three-phase diagram

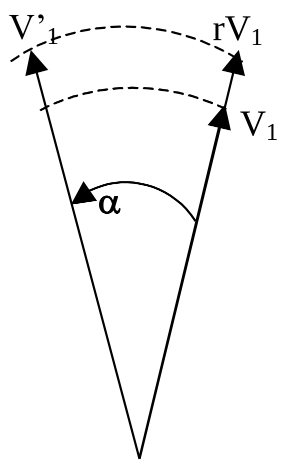
Dual core:



7. In-phase transformer and symmetrical phase shifter

7.1. One phase diagram and equations





$$V2 = \rho V1 e^{j\alpha} - jX(\alpha)I.$$

$$V'1 = r.V1\left(1 + 2j\sin\frac{\alpha}{2}e^{j\frac{\alpha}{2}}\right) = \rho e^{j\alpha}V1$$

r and α vary, and ρ=r

7.2. Expression of the angle and ratio per tap

Same as 4.2 with the addition of the in-phase transformer ratio r.

7.3. Expression of the equivalent series reactance given the angle and the in-phase transformer ratio

Assuming:

- the reactance of the regulating winding varies as the square of the number of turns,
- the equivalent reactance is the sum of the reactance of the in-phase transformer X_r and the reactance of the phase shifter part X_α,
- the phase shifting angle α does not depend on the in-phase ratio r

The equivalent reactance can be written as follows (**not proved**):

$$X(r,\alpha) = X_r(r_{nom})\left(\frac{r}{r_{nom}}\right)^2 + X_\alpha(0) + \left(X_\alpha(\alpha_{\max}) - X_\alpha(0)\right)\left(\frac{\sin(\alpha/2)}{\sin(\alpha_{\max}/2)}\right)^2$$

6 Parameters:

- r_{nom}: nominal ratio of the in-phase transformer
- $X_r(r_{nom})$: equivalent series reactance of the in-phase transformer at nominal in-phase ratio
- α_{max}: maximal phase shift
- $X_\alpha(0)$: equivalent series reactance of the phase shifter part at zero phase shift
- $X_\alpha(\alpha_{max})$: equivalent series reactance of the phase shifter part at maximal phase shift at nominal in-phase ratio (r_{nom})

2 Variables:

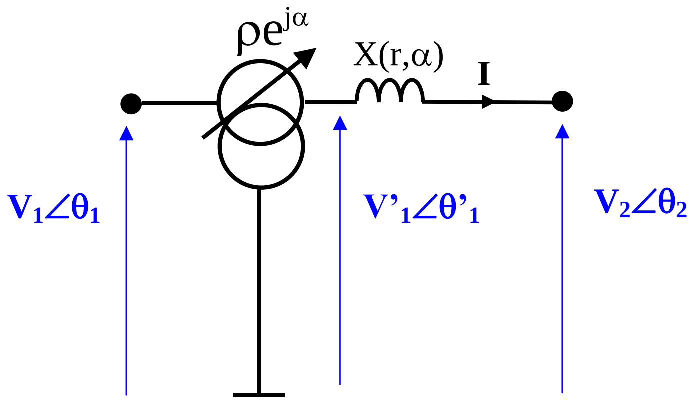
- r: current ratio of the in-phase transformer
- α: current phase shift

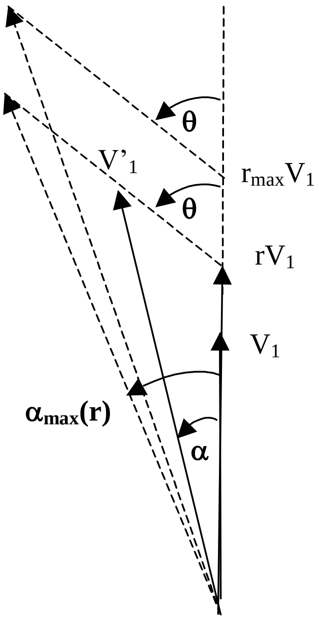
7.4. Technology principles

No example found.

8. In-phase transformer and asymmetrical phase shifter

8.1. One phase diagram and equations





$$V'1 = rV1\left(1 + e^{j\theta}.\frac{\tan\alpha}{\sin\theta - \tan\alpha\cos\theta}\right) = \rho e^{j\alpha}V1 \qquad V2 = \left(V1 - jX(\alpha)I\right).\rho e^{j\alpha}$$

θ is fixed, only r and α are variables, ρ varies as a consequence.

8.2. Expression of the angle and ratio per tap

Not proved.

8.3. Expression of the equivalent series reactance given the angle and the in-phase transformer ratio

<u>Assuming:</u>

- the reactance of the regulating winding varies as the square of the number of turns,
- the equivalent reactance is the sum of the reactance of the in-phase transformer X_r and the reactance of the phase shifter part X_α,

The equivalent series reactance can be written as follows (**not proved**):

$$X(r,\alpha) = X_r(r_{nom})\left(\frac{r}{r_{nom}}\right)^2 + X_\alpha(0) + \left(X_\alpha(\alpha^0_{\max}) - X_\alpha(0)\right)\left(\frac{\tan\alpha}{\tan\alpha_{\max}(r)}\,\frac{\sin\theta - \tan\alpha_{\max}(r)\cos\theta}{\sin\theta - \tan\alpha\cos\theta}\right)^2$$

with $\alpha_{\max}(r) = A\tan\left(\dfrac{\sin\theta}{\dfrac{r}{r_{nom}\tan\alpha^0_{\max}}\left(\sin\theta - \tan\alpha^0_{\max}\cos\theta\right) + \cos\theta}\right)$ with $\alpha^0_{\max} = \alpha_{\max}(r_{nom})$

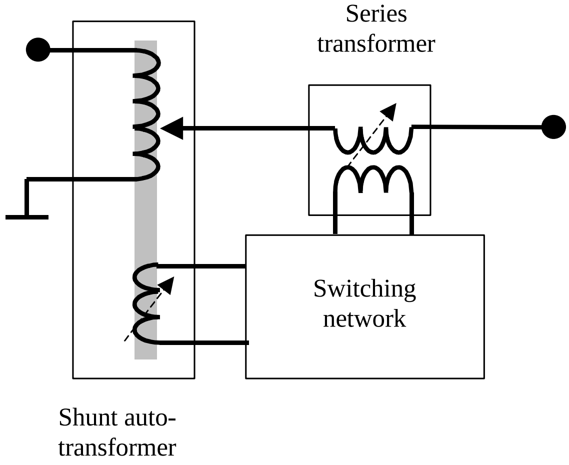
6 Parameters:
- r_{nom}: nominal ratio of the in-phase transformer
- $X_r(r_{nom})$: equivalent series reactance of the in-phase transformer at nominal in-phase ratio r_{nom}
- θ: fix boost voltage angle
- $\alpha^0_{\max} = \alpha_{\max}(r_{nom})$: maximal phase shift for nominal in-phase ratio (r_{nom})
- $X_\alpha(0)$: equivalent series reactance of the phase shifter part at zero phase shift
- $X_\alpha(\alpha^0_{\max})$: equivalent series reactance of the phase shifter part at maximal phase shift at nominal in-phase ratio (r_{nom})

2 Variables:
- r: current ratio of the in-phase transformer
- α: current phase shift

For $\theta = \pi/2$ (quadrature booster): $\alpha_{\max}(r) = A\tan\left(\dfrac{r_{nom}}{r}\tan\alpha^0_{\max}\right)$

8.4. Technology principles

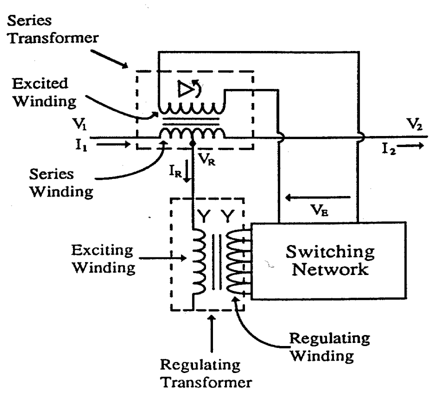
In-phase regulating auto-transformer

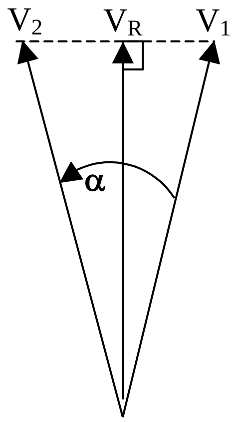


The phase shift regulation may be on the shunt or on the series transformer

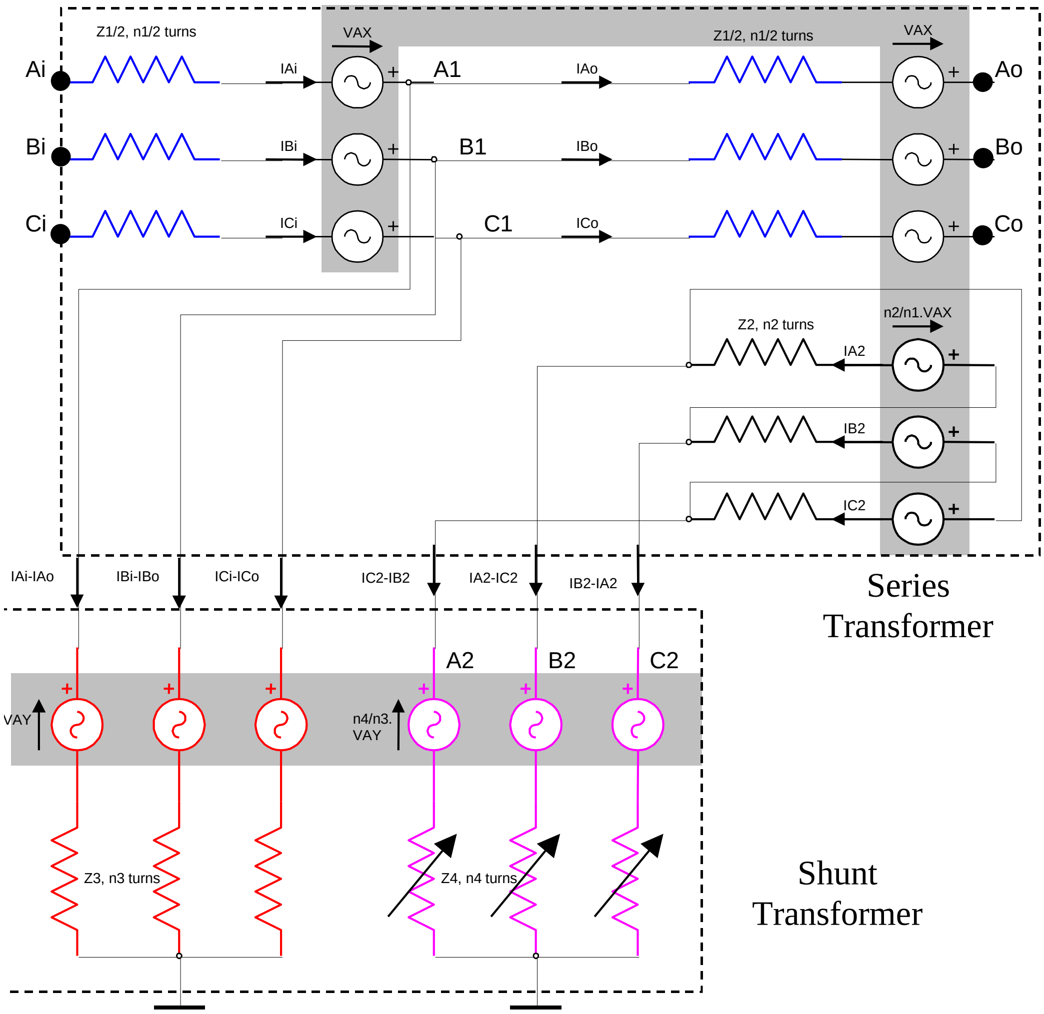
9. Appendix

9.1. Symmetrical Phase Shifters with two cores





Detailed three phase diagram



Only n4 varies.

ENTSO-E AISBL • Avenue de Cortenbergh 100 • 1000 Brussels • Belgium • Tel + 32 2 741 09 50 • Fax + 32 2 741 09 51 • info@entsoe.eu • www. entsoe.eu

Example of numerical values:
n1=120
n2=318
n3=680
$n4_{max}$=182
X1=1,23 Ω
X2=8,64 Ω
X3=101 Ω

Expression of the output current Io and the shunt current I3=Ii-Io:
When considering an ideal phase shifter, the conservation of the electric power is written:
$$Si = 3ViIi^* = 3VoIo^*$$
As the voltage angle is shifted by alpha: $Vo = e^{j\alpha}Vi$
The current angle must also be shifted by alpha as well: $\boxed{Io = e^{j\alpha}Ii}$
Then, the shunt current $\boxed{I3 = Ii - Io = (1-e^{j\alpha})Ii}$

Expression of the shunt equivalent reactance

The shunt reactance Xshunt(α) is defined as the equivalent reactance which crossed by the series input current (*Ii*) would produce the reactive losses of the shunt transformer:
$$Qshunt = 3.Xshunt(\alpha).|Ii|^2$$
with:
$$Qshunt = 3.X3.|I3|^2 + 3.X4.|I4|^2$$
$$|I3|^2 = |Ii|^2|1-e^{j\alpha}|^2 = 2(1-\cos\alpha)|Ii|^2 = 4(\sin(\alpha/2))^2$$
$$|I4|^2 = \frac{n3^2}{n4^2}|I3|^2$$
$$Xshunt(\alpha) = 4(\sin(\alpha/2))^2\left(X3 + X4\left(\frac{n3}{n4}\right)^2\right)$$
Hypothesis: $X4 = X4_{\max}\left(\frac{n4}{n4_{\max}}\right)^2$

Then $Xshunt(\alpha) = 4(\sin(\alpha/2))^2\left(X3 + X4_{\max}\frac{n3^2}{n4^2_{\max}}\right)$

Expression of the equivalent series reactance

The series reactance Xseries(α) is defined as the equivalent reactance which crossed by the series input current (*Ii*) would produce the reactive losses of the series transformer:
$$Qseries = 3.Xseries(\alpha).|Ii|^2$$
with:
$$Qseries = 3.\frac{X1}{2}.|Ii|^2 + 3.\frac{X1}{2}.|Io|^2 + 3.X2.|I2|^2$$

ENTSO-E AISBL • Avenue de Cortenbergh 100 • 1000 Brussels • Belgium • Tel + 32 2 741 09 50 • Fax + 32 2 741 09 51 • info@entsoe.eu • www. entsoe.eu

as seen previously: $Io = e^{j\alpha} Ii$

series transformer current relationship: $\frac{n1}{2}.Ii + \frac{n1}{2}.Io = +n2.I2$

then $I2 = \frac{n1}{2n2}\left(1+e^{j\alpha}\right)Ii$ and $\left|I2\right|^2 = \left(\frac{n1}{2n2}\right)^2 \left|Ii\right|^2 \left|1+e^{j\alpha}\right|^2 = \frac{1}{2}\frac{n1^2}{n2^2}(1+\cos\alpha)\left|Ii\right|^2$

then

$$Xseries(\alpha) = X1 + \frac{1}{2}\left(\frac{n1}{n2}\right)^2 (1+\cos\alpha)X2 = X1 + \left(\frac{n1}{n2}\right)^2 (1-\sin(\alpha/2))X2$$

$$Xseries(\alpha) = \left(X1 + \left(\frac{n1}{n2}\right)^2 X2\right) - \left(\frac{n1}{n2}\right)^2 X2\left(\sin(\alpha/2)\right)^2$$

Expression of the total equivalent reactance X

$$X(\alpha) = \left(X1 + \left(\frac{n1}{n2}\right)^2 X2\right) + 4\left(\sin(\alpha/2)\right)^2 \left(X3 + X4_{\max}\frac{n3^2}{n4^2_{\max}} - \left(\frac{n1}{2n2}\right)^2 X2\right)$$

or

$$X(\alpha) = X(0) + \left(X(\alpha_{\max}) - X(0)\right)\left(\frac{\sin(\alpha/2)}{\sin(\alpha_{\max}/2)}\right)^2$$

with:

$$X(0) = \left(X1 + \left(\frac{n1}{n2}\right)^2 X2\right)$$

$$X(\alpha_{\max}) - X(0) = 4\left(\sin(\alpha_{\max}/2)\right)^2 \left(X3 + X4_{\max}\frac{n3^2}{n4^2_{\max}} - \left(\frac{n1}{2n2}\right)^2 X2\right)$$

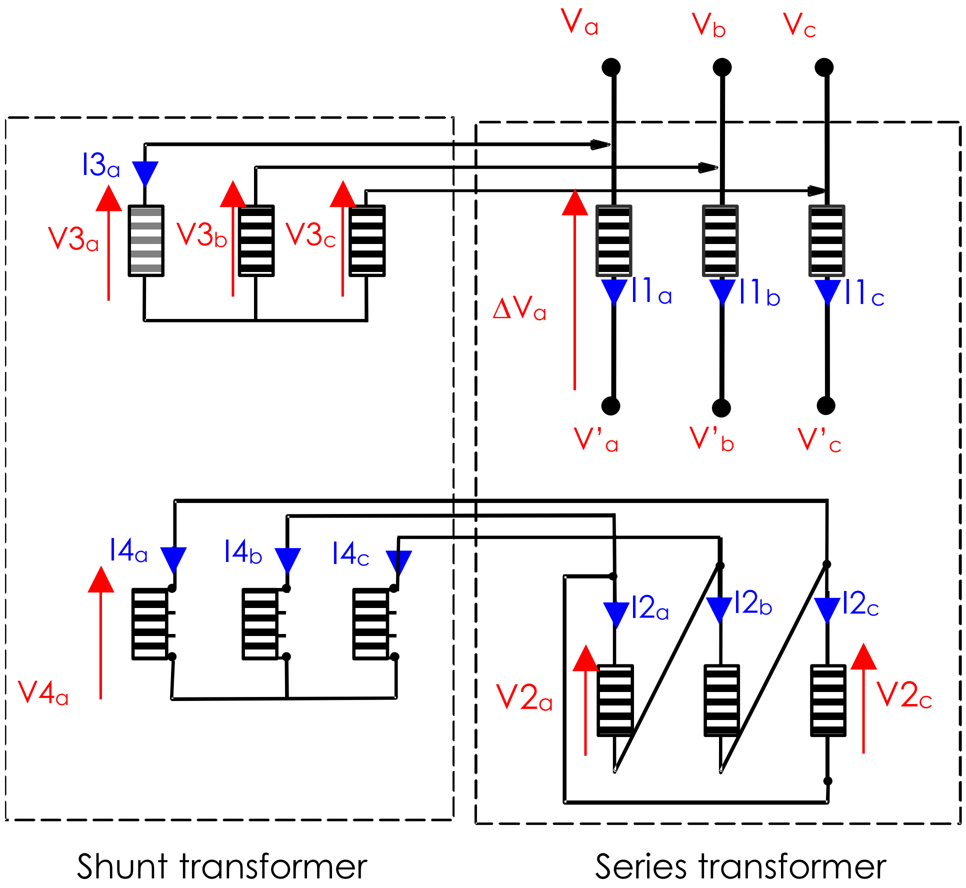
Remark: only α varies.

ENTSO-E AISBL • Avenue de Cortenbergh 100 • 1000 Brussels • Belgium • Tel + 32 2 741 09 50 • Fax + 32 2 741 09 51 • info@entsoe.eu • www. entsoe.eu

9.2. Quadrature boosters

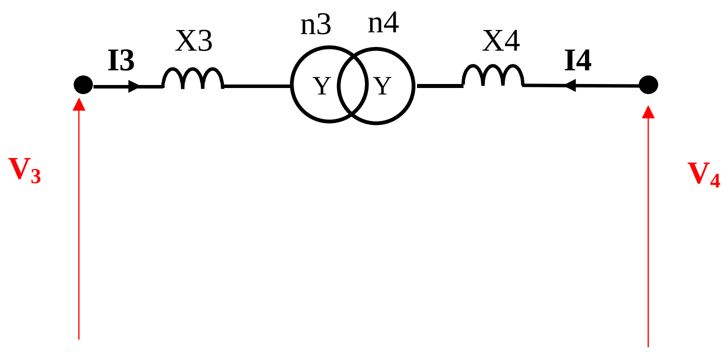
9.3. Quadrature booster with two cores

Detailed three phase diagram



For each physical value X (X=V or I) $X = X_a = aX_b = a^2 X_c$ with $a = e^{j\frac{2\pi}{3}}$

shunt transformer with variable ratio n4/n3 with X3 reactance on the primary winding and X4 on the secondary winding:



$$V4 - jX4.I4 = \frac{n4}{n3}\left(V3 - jX3.I3\right) \qquad I3 = -\frac{n4}{n3} I4$$

$$V4 = \frac{n4}{n3}V3 + j\left(X_4 + X_3\left(\frac{n4}{n3}\right)^2\right)I4$$

Hypothesis: the X4 reactance varies like the square of the number of turns n4:

$$X_4 = X_4^{\max}\left(\frac{n4}{n4_{\max}}\right)^2$$

Series transformer of fixed ratio n1/n2 with X1 reactance on the primary winding and X2 on the secondary winding:

$$\Delta V = \frac{n1}{n2}V2 + j\left(X1 + \left(\frac{n1}{n2}\right)^2 X2\right)I1$$

$$I2 = -\frac{n1}{n2}I1$$

Link between series and shunt transformers:

- Primary windings:
  $$V3 = V$$
- Secondary windings:
  $$I4 = I4_a = I2_b - I2_c = (a^2 - a)I2_a = -j\sqrt{3}I2$$
  $$V2 = V2_a = V4_b - V4_c = (a^2 - a)V4 = -j\sqrt{3}V4$$

<u>Calculation:</u>

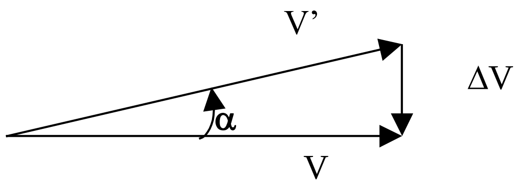
$$\Delta V = -j\frac{n4.n1}{n3.n2}\sqrt{3}V + j\left(X1 + \left(\frac{n1}{n2}\right)^2 X2 + 3\left(\frac{n1}{n2}\right)^2 n4^2\left(\frac{X4^{\max}}{n4_{\max}^2} + \frac{X3}{n3^2}\right)\right)I1$$

$$I3 = -j\frac{n4.n1}{n3.n2}\sqrt{3}I1$$

<u>At no load conditions (I=0):</u>

$$\Delta V = -j\frac{n4.n1}{n3.n2}\sqrt{3}V$$

And geometrically:



$$\Delta V = -jV\tan\alpha$$

hence $\dfrac{n4}{n4^{\max}} = \dfrac{\tan\alpha}{\tan\alpha_{\max}}$

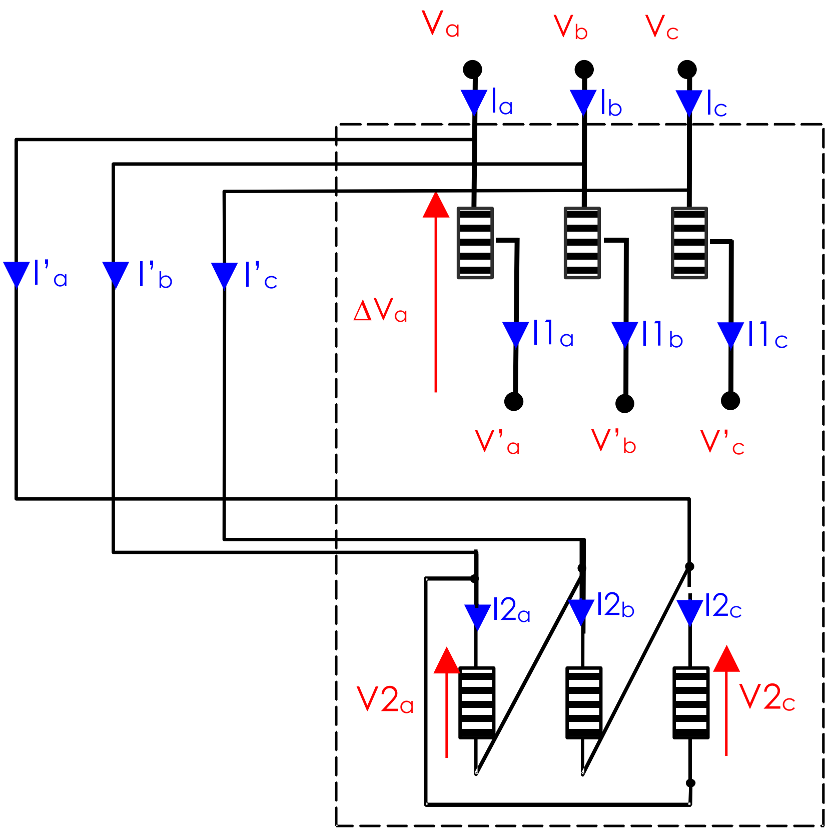
And $\Delta V = -j\frac{n4.n1}{n3.n2}\sqrt{3}V + j\left(X1 + \left(\frac{n1}{n2}\right)^2 X2 + 3n4_{max}^2\left(\frac{n1}{n2}\right)^2\left(\frac{X4^{max}}{n4_{max}^2} + \frac{X3}{n3^2}\right)\left(\frac{\tan\alpha}{\tan\alpha_{max}}\right)^2\right)I1$

comes:

$$\Delta V = -j\frac{n4.n1}{n3.n2}\sqrt{3}V + jXI \quad \text{with} \quad X(\alpha) = X(0) + \left(X(\alpha_{max}) - X(0)\right)\left(\frac{\tan(\alpha)}{\tan(\alpha_{max})}\right)^2$$

9.4. Quadrature booster with a single core

Detailed three phase diagram



n1 varies.

$$I' = j\sqrt{3}I2$$
$$V2 = -j\sqrt{3}V$$
$$\Delta V = \frac{n1}{n2}V2 + j\left(X1 + \left(\frac{n1}{n2}\right)^2 X2\right)I1$$

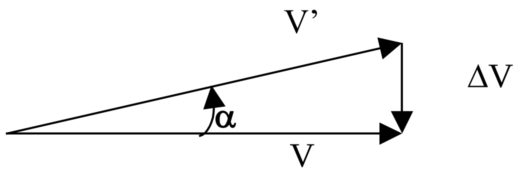
$$\Delta V = -j\sqrt{3}\frac{n1}{n2}V + j\left(X1 + \left(\frac{n1}{n2}\right)^2 X2\right)I1$$

Assuming $X1 = X1_{\max}\left(\frac{n1}{n1_{\max}}\right)^2$

$$\Delta V = -j\sqrt{3}\frac{n1}{n2}V + j\left(\frac{X1_{\max}}{n1_{\max}} + \frac{X2}{n2^2}\right)n1^2 I1$$

At no-load: $\Delta V = -j\sqrt{3}\frac{n1}{n2}V$

And geometrically:



$$\Delta V = -jV\tan\alpha$$

hence $\frac{n1}{n1^{\max}} = \frac{\tan\alpha}{\tan\alpha_{\max}}$

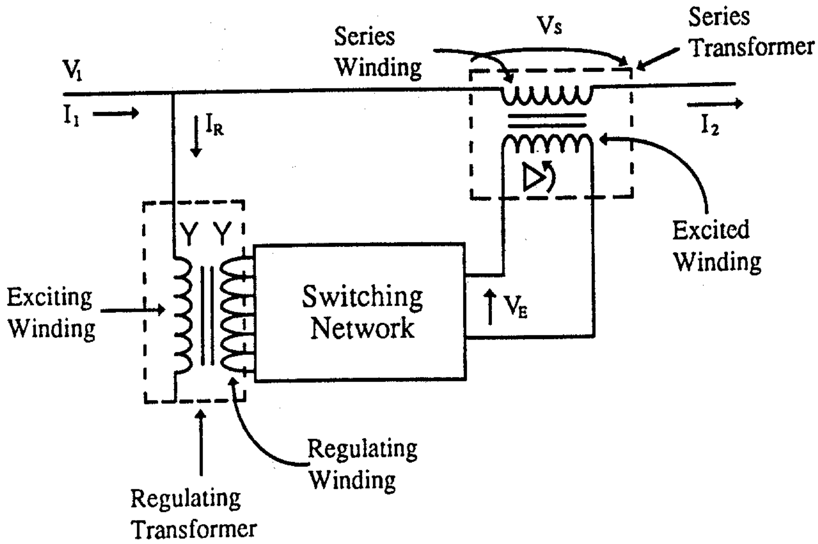
$$X(\alpha) = \left(X1_{\max} + X2\frac{n1_{\max}^2}{n2^2}\right)\left(\frac{\tan\alpha}{\tan\alpha_{\max}}\right)^2$$

$$X(\alpha) = X(\alpha_{\max})\left(\frac{\tan\alpha}{\tan\alpha_{\max}}\right)^2$$

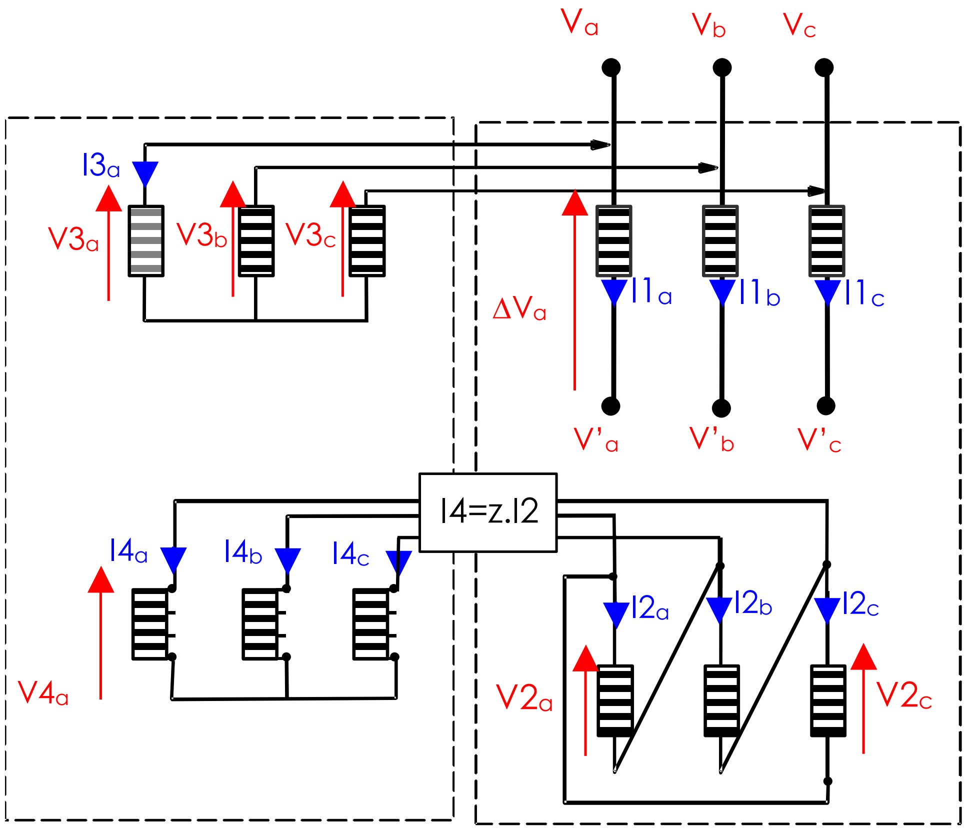
ENTSO-E AISBL • Avenue de Cortenbergh 100 • 1000 Brussels • Belgium • Tel + 32 2 741 09 50 • Fax + 32 2 741 09 51 • info@entsoe.eu • www. entsoe.eu

9.5. Asymmetrical phase shifter

9.6. Asymmetrical phase shifter with two cores



Detailed three phase diagram:



Shunt transformer Series transformer

For each physical value X (X=V or I) $X = X_a = aX_b = a^2 X_c$ with $a = e^{j\frac{2\pi}{3}}$

shunt transformer equations:

$$V4 = \frac{n4}{n3}V3 + j\left(X_4 + X_3\left(\frac{n4}{n3}\right)^2\right)I4$$

$$I3 = -\frac{n4}{n3}I4$$

Hypothesis: the X_2 reactance varies like the square of the k_1 ratio:

$$X_4 = X_4^{\max}\left(\frac{n4}{n4_{\max}}\right)^2$$

Series transformer of fixed ratio k_2 with X_{series} reactance on the primary winding:

$$\Delta V = \frac{n1}{n2}V2 + j\left(X1 + \left(\frac{n1}{n2}\right)^2 X2\right)I1$$

$$I2 = -\frac{n1}{n2}I1$$

Link between series and shunt transformers:

- Primary windings:
  $V3 = V$
- “z”, a complex number, determines the coupling between the series and shunt transformer secondary windings:
  $I4 = I4_a = z.I2$
  $V2 = z.V4$
  for a quadrature booster $z = -j\sqrt{3}$

<u>Calculation:</u>

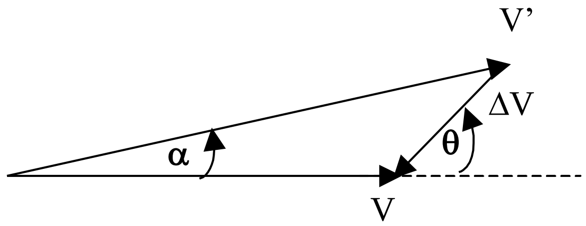
$$\Delta V = \frac{n4.n1}{n3.n2}.z.V + j\left(X1 + \left(\frac{n1}{n2}\right)^2 X2 - z^2\left(\frac{n1}{n2}\right)^2 n4^2\left(\frac{X4^{\max}}{n4^2_{\max}} + \frac{X3}{n3^2}\right)\right)I1$$

$$I3 = \frac{n4.n1}{n3.n2}.z.I1$$

<u>At no load conditions (I=0):</u>

$$\Delta V = \frac{n4.n1}{n3.n2}.z.V$$

And geometrically: $\Delta V = -e^{j\theta}.V\frac{\tan\alpha}{\sin\theta - \tan\alpha\cos\theta}$



hence $$\frac{n4}{n4^{max}} = \frac{\tan\alpha}{\tan\alpha_{max}} \frac{\sin\theta - \tan\alpha_{max}\cos\theta}{\sin\theta - \tan\alpha\cos\theta}$$

And

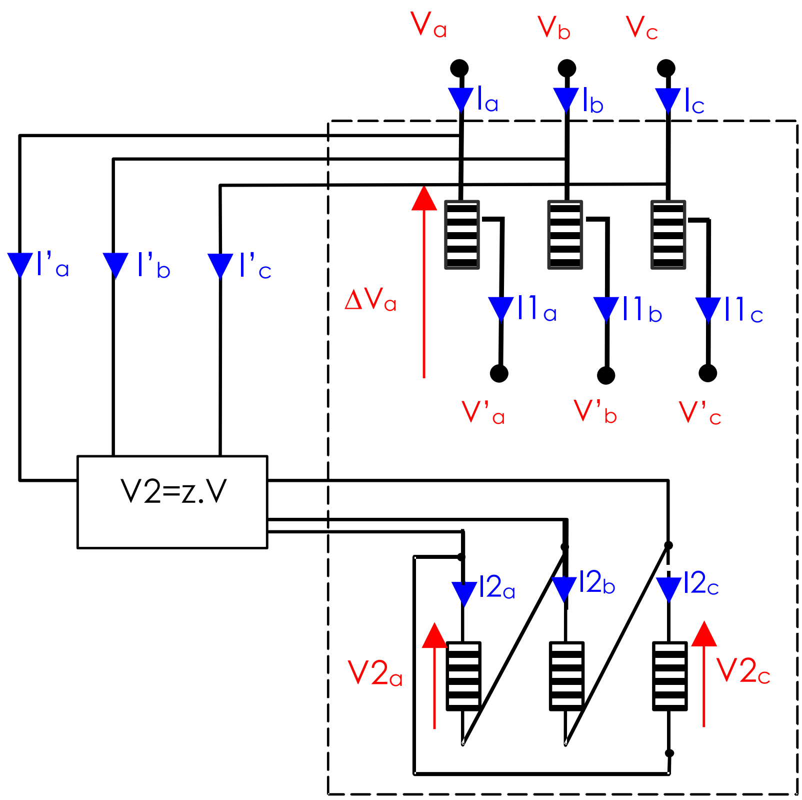
$$\Delta V = -j\frac{n4.n1}{n3.n2}\sqrt{3}V + j\left(X1 + \left(\frac{n1}{n2}\right)^2 X2 - z^2 n4_{max}^2 \left(\frac{n1}{n2}\right)^2 \left(\frac{X4^{max}}{n4_{max}^2} + \frac{X3}{n3^2}\right)\left(\frac{\tan\alpha}{\tan\alpha_{max}} \frac{\sin\theta - \tan\alpha_{max}\cos\theta}{\sin\theta - \tan\alpha\cos\theta}\right)^2 \right) I1$$

comes:

$$\Delta V = z\frac{n4.n1}{n3.n2}V + jXI$$ with

$$X(\alpha) = X(0) + \left(X(\alpha_{max}) - X(0)\right)\left(\frac{\tan\alpha}{\tan\alpha_{max}} \frac{\sin\theta - \tan\alpha_{max}\cos\theta}{\sin\theta - \tan\alpha\cos\theta}\right)^2$$

9.7. Asymmetrical phase shifter with a single core



ENTSO-E AISBL • Avenue de Cortenbergh 100 • 1000 Brussels • Belgium • Tel + 32 2 741 09 50 • Fax + 32 2 741 09 51 • info@entsoe.eu • www. entsoe.eu

Only n1 varies.

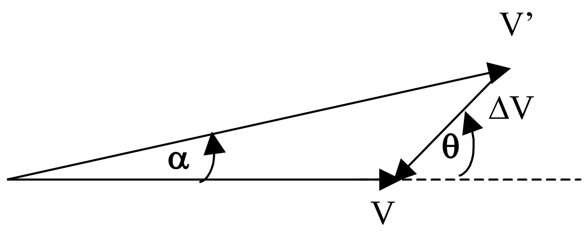
$$I' = -z.I2$$
$$V2 = z.V$$
$$\Delta V = \frac{n1}{n2}V2 + j\left(X1 + \left(\frac{n1}{n2}\right)^2 X2 \right) I1$$
$$\Delta V = z.\frac{n1}{n2}V + j\left(X1 + \left(\frac{n1}{n2}\right)^2 X2 \right) I1$$

Assuming $X1 = X1_{max}\left(\frac{n1}{n1_{max}}\right)^2$

$$\Delta V = z.\frac{n1}{n2}V + j\left(\frac{X1_{max}}{n1_{max}} + \frac{X2}{n2^2} \right) n1^2\, I1$$

At no-load: $\Delta V = z.\frac{n1}{n2}V$

And geometrically: $\Delta V = -e^{j\theta}.V\frac{\tan\alpha}{\sin\theta - \tan\alpha\cos\theta}$



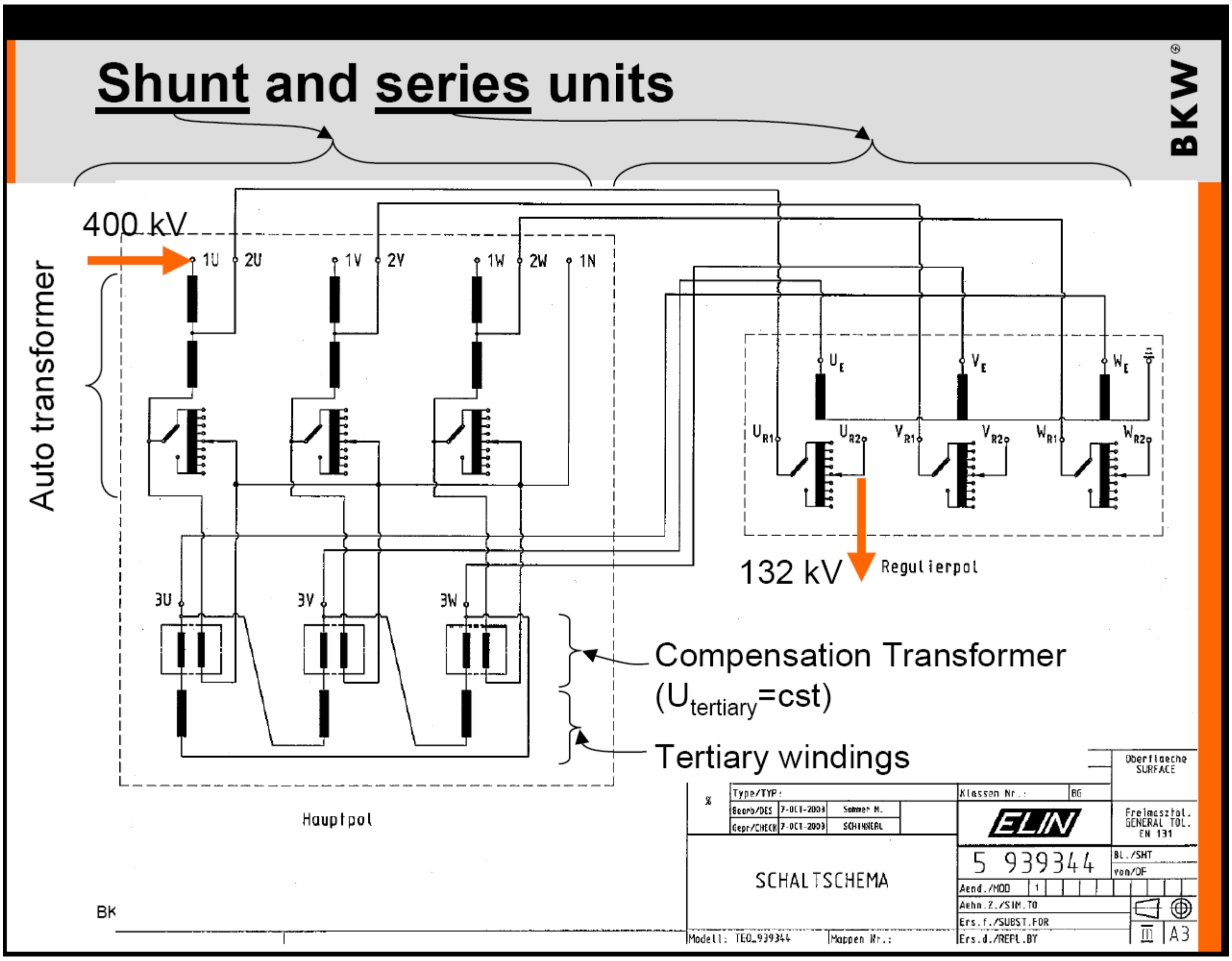
hence $\frac{n1}{n1_{max}} = \frac{\tan\alpha}{\tan\alpha_{max}}\frac{\sin\theta - \tan\alpha_{max}\cos\theta}{\sin\theta - \tan\alpha\cos\theta}$

$$X(\alpha) = \left(X1_{max} + X2\frac{n1_{max}^2}{n2^2} \right)\left(\frac{\tan\alpha}{\tan\alpha_{max}}\frac{\sin\theta - \tan\alpha_{max}\cos\theta}{\sin\theta - \tan\alpha\cos\theta} \right)^2$$
$$X(\alpha) = X(\alpha_{max})\left(\frac{\tan\alpha}{\tan\alpha_{max}}\frac{\sin\theta - \tan\alpha_{max}\cos\theta}{\sin\theta - \tan\alpha\cos\theta} \right)^2$$

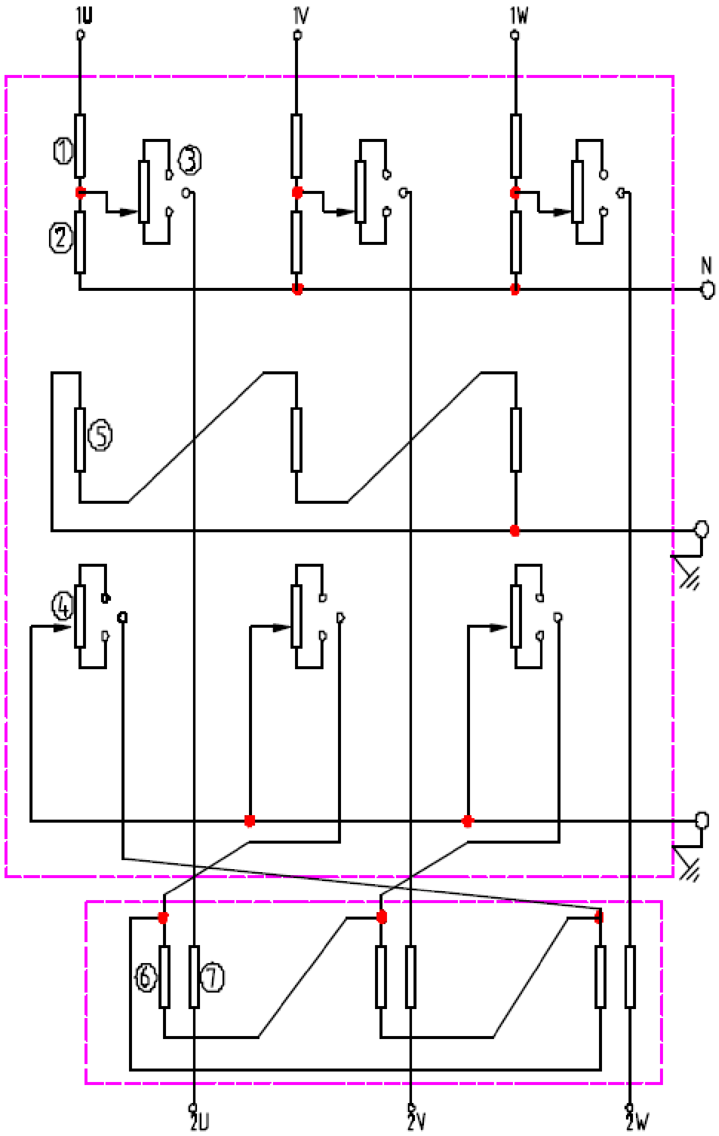
ENTSO-E AISBL • Avenue de Cortenbergh 100 • 1000 Brussels • Belgium • Tel + 32 2 741 09 50 • Fax + 32 2 741 09 51 • info@entsoe.eu • www. entsoe.eu

9.8. In-phase transformer and asymmetrical phase shifter

Examples of detailed three-phase diagrams: voltage regulating auto-transformer and Quadrature booster:



Voltage regulating auto-transformer and Quadrature booster:



1 EHV winding
2 HV winding
3 EV voltage regulating winding
4 Phase shift regulating winding
5 Tertiary winding
6 Primary winding of the series transformer
7 Secondary winding of the series transformer

ENTSO-E AISBL • Avenue de Cortenbergh 100 • 1000 Brussels • Belgium • Tel + 32 2 741 09 50 • Fax + 32 2 741 09 51 • info@entsoe.eu • www. entsoe.eu